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Engineering Sciences Division
(Solid Mechanics Program: Dr. Gary Anderson)

on project

# VOID NUCLEATION IN NONLINEAR SOLID MECHANICS

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from

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Consider a solid sphere of unit radius subjected to a uniformly-distributed radial tensile traction p on its surface. In response to this loading, the body can expand into a larger solid sphere. However for certain materials, when p is sufficiently large, it is energetically more favorable for the body to instead develop an internal spherical cavity. A bifurcation analysis yields the value $p_{cr}$ of the applied stress at which this instability occurs, without the need for an ad hoc failure criterion, Ball(1982).

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This phenomenon of internal rupture is closely related to the well-known phenomenon of cavitation in fluids, and has therefore been dubbed "cavitation" even in solids. Since the nucleation of a void in a stressed solid is often a precursor to failure, it would be useful to be able to predict the conditions under which cavitation occurs.

The present project has been aimed at trying to determine whether cavitation, viewed as an intrinsic material instability, provides insight into the phenomenon of void nucleation. The project had two principal goals: one, to better understand the phenomenon of spherically (of cylindrically) symmetric cavitation, and two, to study the phenomenon of cavitation in circumstances that do not possess such symmetry. This report contains the results of the investigation.

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SCIENTIFIC PERSONNEL SUPPORTED:

a) Rohan Abeyaratne (Principal investigator)
b) Hang-sheng Hou (PhD graduate student)

DEGREES AWARDED OR FORTHCOMING:

a) Hang-sheng Hou, Cavitation Instability in Solids, PhD expected October 1990, Massachusetts Institute of Technology.

LIST OF PAPERS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP:

1) R. Abeyaratne and J.K. Knowles , Non-elliptic elastic materials and the modeling of dissipative mechanical behavior: an example, Journal of Elasticity, Vol. 18, (1987), pp. 227-278.

2) D.T. Chung, C.O. Horgan and R. Abeyaratne, A note on a bifurcation problem in finite plasticity related to void nucleation, International Journal of Solids and Structure, Vol. 23, (1987), pp. 983-988.

3) N. Ertan, Influence of compressibility and hardening on cavitation, ASCE Journal of Engineering Mechanics, Vol. 114, (1988), pp. 1231-1244.

4) R. Abeyaratne and H-s. Hou, Growth of an infinitesimal cavity in a rate-dependent solid, ASME Journal of Applied Mechanics, Vol. 56, (1989), pp. 40-46.

5) R. Stringfellow and R. Abeyaratne, Cavitation in an elastomer: comparison of theory with experiment, Journal of Materials Science and Engineering, Vol. A112, (1989), pp. 127-131.

6) R. Abeyaratne and H-s. Hou, Void collapse in an elastic solid, accepted for publication in the Journal of Elasticity, July 1989.

7) H-s. Hou and Y. Zhang, The effect of axial stretch on cavitation, accepted for publication in the International Journal of Nonlinear Mechanics, November 1989.

8) R. Abeyaratne and H-s. Hou, On the occurrence of the cavitation instability relative to the asymmetric instability under symmetric dead load conditions, submitted to Quarterly Journal of Mechanics and Applied Mathematics, April 1990.

LIST OF PAPERS CURRENTLY UNDER PREPARATION UNDER ARO SPONSORSHIP:

9) H-s. Hou and R. Abeyaratne, Axi-symmetric cavitation in solids, manuscript in preparation.

10) H-s. Hou and R. Abeyaratne, On the cavitated instability and its relationship to a continuum of local asymmetric configurations, manuscript in preparation.

PRESENTATIONS:

1) Rohan Abeyaratne, Brown University Solid Mechanics Seminar, Providence, Rhode Island, March 1987.

2) Rohan Abeyaratne, Harvard University Applied Mechanics Seminar, Cambridge, Massachusetts, April 1987.

3) Hang-sheng Hou and Rohan Abeyaratne, "Growth of a void in a rate-dependent material", 24th Annual Meeting of Society of Engineering Science, Salt Lake City, Utah, September 1987.

4) Rohan Abeyaratne, University of Minnesota Engineering Mechanics Seminar, Minneapolis, Minnesota, October 1987.

5) Rohan Abeyaratne, Texas A and M University Engineering Mechanics Seminar, College Station, Texas, November 1987.

6) Rohan Abeyaratne, University of Texas Engineering Mechanics Seminar, Austin, Texas, November 1987.

7) Rohan Abeyaratne, University of Houston Mechanical Engineering Seminar, Houston, Texas, March 1988.

8) Rohan Abeyaratne, "On the formulation of problems in nonlinear elasticity", Workshop on the Reliability of the Finite Element Method, College Park, Maryland, April 1988.

9) Rohan Abeyaratne, University of Pittsburgh Applied Mechanics Seminar, Pittsburgh, Pennsylvania, October 1988.

10) Rohan Abeyaratne, Massachusetts Institute of Technology Applied Mechanics Seminar, Cambridge, Massachusetts, October 1988.

11) Rohan Abeyaratne, Michigan State University Mechanics Seminar, East Lansing, Michigan, November 1988.

12) Hang-sheng Hou and Rohan Abeyaratne, "Spherically symmetric void collapse in an incompressible elastic solid", 21st Midwest Mechanics Conference, Houghton, Michigan, August 1989.

13) Hang-sheng Hou and Rohan Abeyaratne, "Void collapse under bi-axial loading in a class of elastic solids", 26th Annual Meeting of the Society of Engineering Science, Ann Arbor, Michigan, September 1989.

14) Rohan Abeyaratne, Cornell University Theoretical and Applied Mechanics Seminar, Ithaca, New York, March 1989.

15) Rohan Abeyaratne, The Johns Hopkins University Mechanical Engineering Seminar, Baltimore, Maryland, April 1990.

## CONTACT WITH ARMY PERSONNEL

Dr. Roshdy Barsoum, of the Army Materials Technology Laboratory in Watertown, Massachusetts, visited with me at MIT on April 28, 1987 and on November 8, 1988.

Claudia Quigley, of the Army Materials Technology Laboratory in Watertown, Massachusetts, worked on a project on the bulging of compressed rubber cylinders under my supervision, during Fall 1987. I had continued discussions with her on the fracture of elastomers during the Fall and Spring of 1988.

Visited the Army Materials Technology Laboratory in Watertown, Massachusetts, and had meeting with Dr. Tony Chou, Spring 1989.

Visited, a couple of times, and had discussions on a number of additional occasions, with Dr. Tusit Weerasooriya of the Army Materials Technology Laboratory in Watertown, Massachusetts, about issues in large strain plasticity, and also about the possibility of conducting experiments on shape memory alloys at MTL, Spring 1989 - onwards.

Had some contact with Dr. Tim Wright of the Army Proving Ground in Aberdeen when I visited Johns Hopkins University to present a seminar, and again at the U.S. National Congress of Mechanics, Spring 1990.

DESCRIPTION OF PROBLEM AND RESULTS:

Consider a solid sphere of unit radius subjected to a uniformly-distributed radial tensile traction p on its surface. In response to this loading, the body can expand into a larger solid sphere. However for certain materials, when p is sufficiently large, it is energetically more favorable for the body to instead develop an internal spherical cavity. A bifurcation analysis yields the value $p_{cr}$ of the applied stress at which this instability occurs, without the need for an ad hoc failure criterion, Ball(1982).

This phenomenon of internal rupture is closely related to the well-known phenomenon of cavitation in fluids, and has therefore been dubbed "cavitation" even in solids. Since the nucleation of a void in a stressed solid is often a precursor to failure, it would be useful to be able to predict the conditions under which cavitation occurs.

The present project has been aimed at trying to determine whether cavitation, viewed as an intrinsic material instability, provides insight into the phenomenon of void nucleation. The project had two principal goals: one, to better understand the phenomenon of spherically (of cylindrically) symmetric cavitation, and two, to study the phenomenon of cavitation in circumstances that do not possess such symmetry.

PART A: RADIALLY SYMMETRIC CAVITATION

The effect of compressibility and hardening on cavitation:
The study of cavitation in compressible materials is considerably more difficult than the corresponding problem for incompressible materials because of the difficulty involved in solving the associated nonlinear differential equation. We studied this problem for a special class of compressible elastic materials, the material model being a generalization of one for foam rubber, and the aim being to examine the qualitative influence of material compressibility on cavitation. Specifically, a bifurcation problem for a solid circular cylinder composed of this elastic material was studied. The curved surface of the cylinder was subjected to a radial stretch $\lambda$ (>1), and the cylinder was in a state of plane strain. A cylindrical cavity, coaxial with the cylinder, is found to emerge when $\lambda$ reaches a critical value (say $\lambda_{cr}$) at which the homogeneous deformation becomes unstable. We find that $\lambda_{cr}$ decreases as the material becomes softer and also as it becomes less compressible. The corresponding value of radial true stress $\tau_{cr}$ also decreases as the material becomes softer, but increases as it becomes less compressible. Figures 1 and 2 show this variation.

Reference: N. Ertan, Influence of compressibility and hardening on cavitation, ASCE Journal of Engineering Mechanics, Vol. 114, (1988), pp. 1231-1244.

The effect of rate-dependence on the growth of an infinitesimal cavity:
In this study we examined the effect of rate-dependence on the phenomenon of cavitation in an incompressible material. Specifically, a sphere containing a traction-free void of infinitesimal initial radius was considered, and the outer surface of the sphere was subjected to a prescribed uniform radial nominal stress p, which was suddenly applied and then held constant. The sphere was composed of a particular class of incompressible rate-dependent materials. The large strains which occur in the vicinity of the void were accounted for in the

analysis and the problem was reduced to a nonlinear initial-value problem, which was then studied qualitatively through a phase-plane analysis. The principal result derived in this study is characterized by two equations relating the applied stress p and the current cavity radius b: $p = \hat{p}(b)$ and $p = \bar{p}(b)$. The first of these describes a curve that separates the (p,b)-plane into regions where cavitation does and does not occur. The second describes a curve which subdivides the former subregion -- the post-cavitation region -- into domains where void growth occurs slowly and rapidly. Figure 3 shows an example of these curves for a particular set of material parameters.

Reference: R. Abeyaratne and H-s. Hou, Growth of an Infinitesimal Cavity in a Rate-Dependent Solid, ASME Journal of Applied Mechanics, Vol. 56, (1989), pp. 40-46.

<u>The effect of axial stretch on cavitation in an elastic cylinder</u>

In this study the phenomenon of cavitation in an elastic cylinder subjected to a combined axial stretch $\lambda_z$ and a radial traction p was examined. One possible response of the cylinder to this loading, for all $\lambda_z$ and p, is a pure homogeneous deformation. However, for some materials, the homogeneous deformation becomes unstable at certain critical values of the pair $(\lambda_z, p)$ and a second deformation involving a cylindrical cavity emerges. We determined the values of $(\lambda_z, p)$ at which this happens. The results are displayed by constructing a curve in the $(\lambda_z, p)$-plane which divides this plane into regions where the homogeneous deformation is stable and unstable.

From the results of this study we have observed that a material which does not exhibit cavitation at one value of the axial stretch $\lambda_z$ might do so when it is subjected to some other value of $\lambda_z$. We also observed that a material which, at some $\lambda_z$, possesses cavitated deformations which are unstable, could exhibit stable cavitated deformations when a different axial stretch is imposed. Figure 4 shows the critical curves in the $(\lambda_z, p)$-plane for different values of material hardening.

Reference: H-s. Hou and Y. Zhang, The effect of axial stretch on cavitation, accepted for publication in the International Journal of Nonlinear Mechanics, November 1989.

<u>Comparison of analysis with experiments</u>

Gent and Lindley(1958) conducted experiments on thin disks of vulcanized rubber by bonding its faces to two stiff metal disks, and then applying uniaxial loads to the metal disks normal to its faces. They observed direct and indirect evidence of flaw initiation at the center of the rubber specimen when the load reached a critical value. By repeating the experiment on a series of specimens which had been pre-treated differently, they were able to relate the critical load at void initiation to the Young's modulus of the material.

Oberth and Bruenner(1965) conducted tensile tests on a standard specimen of polyurethane in which they had embedded a small metallic sphere; the sphere was well bonded to the surrounding matrix material. During the tension tests they observed the appearance of small cavities (near the poles of the spherical par-

ticle) when the load reached a critical value. They too repeated the tests on a series of specimens that had been pre-treated differently, and were able to relate the Young's modulus of the material to the critical load at void initiation.

We carried out a finite element analysis of the two test specimens used in these experiments, and used the results of these numerical studies, in conjunction with the predictions of a cavitation analysis, to predict theoretically the load when cavitation occurs; in both cases, the results were in striking agreement with the experimental observations.

Reference: R. Stringfellow and R. Abeyaratne, Cavitation in an elastomer: comparison of theory with experiment, Journal of Materials Science and Engineering, Vol. A112, (1989), pp. 127-131.

Void collapse

In this study we examined the collapse of a void, the phenomenon in which an existing void closes due to an instability. This is the opposite phenomenon to cavitation. A complete analysis of the collapse of a void in an incompressible elastic material was carried out in the special cases of spherical and axial symmetry . The analysis predicts the critical load at which collapse occurs.

We considered large, radially symmetric, deformations of an infinite medium composed of an isotropic, incompressible elastic material surrounding a traction-free spherical cavity. The body is subjected to a uniform pressure at infinity and we examine the possibility of void collapse, i.e. the possibility that the void radius becomes zero at a finite value of the applied stress. This does not occur in all materials. We have determined the complete sub-class of incompressible elastic materials that do exhibit this phenomenon, and for such materials, found the value of the applied load at which collapse occurs. We have also examined the stability of the deformation, both from minimum potential energy and dynamic stability points of view. The results were illustrated by considering a particular class of power-law materials. Figure 5 shows the variation of the cavity radius b with the applied pressure p for three values of the hardening exponent.

In certain respects, the results for void collapse are complementary to those for void expansion. In this complementary problem, one is interested in examining if, and when, a cavity in an infinite medium can grow without bound. Many formal similarities exist between the results of the analyses of these two problems.

Reference: R. Abeyaratne and H-s. Hou, Void collapse in an incompressible elastic solid, accepted for publication in the Journal of Elasticity, July 1989.

The relationship between local and global instabilities during cavitation.

Consider a solid sphere subjected to a uniform radial tensile load, which, at some critical value of the load, bifurcates into a configuration involving a concentric internal cavity. Consider an infinitesimal cube of material within this body, just prior to cavitation, which is oriented with its edges parallel to the radial and hoop directions. In this configuration, this material element is subjected to a deformation with equal principal stretches $\lambda_1 = \lambda_2 = \lambda_3$ and a

hydrostatic state of stress. Immediately after cavitation, this same material element has the geometric shape of a plate with $\lambda_1 < 1$ and $\lambda_2 = \lambda_3 > 1$ (where $\lambda_1$ is the radial stretch and $\lambda_2$, $\lambda_3$ are the hoop stretches); see Figure 6. Thus it is apparent that when the body undergoes the (global) cavitation instability, each infinitesimal material element undergoes this local "cube → plate" instability (which we refer to as an asymmetric instability). This suggests that cavitation might be viewed as the occurrence of a continuum of local asymmetric instabilities, so that

i) a material that exhibits the cavitation instability must necessarily exhibit the asymmetric instability;

ii) the critical load at cavitation must be a suitable average of a sequence of critical loads for the asymmetric instability;

iii) a material that exhibits stable cavitation must necessarily exhibit stable asymmetric deformations.

We have been able to prove these results, thereby allowing us to understand the intrinsic nature of the cavitation instability in terms of a more simple instability that occurs locally.

Reference: H-s. Hou and R. Abeyaratne, On the cavitated instability and its relationship to a continuum of local asymmetric configurations?, manuscript in preparation.

## PART B: NON-RADIALLY SYMMETRIC CAVITATION

Most previous studies on cavitation, including those described in Part A above, have been restricted to the case of radial (i.e. spherical or cylindrical) symmetry. Many practical situations do not possess such symmetry and it is therefore important to extend the radially-symmetric studies to more general situations. The particular three-dimensional case of axial-symmetry, where the nucleated void is, roughly, ellipsoidal in shape, is a special case of some importance.

Consider an incompressible, isotropic medium, subjected to a remotely applied axially-symmetric state of stress; let $\tau_1$ denote the axial stress and $\tau_2$ each of the two equal in-plane stresses. The body deforms in an axially-symmetric manner. If $\tau_1 = \tau_2$, we have spherical symmetry and an (infinitesimal) spherical void appears at the origin, when the applied stress reaches the value $\tau_1 = \tau_2 = \tau_{cr}$. In the general case ($\tau_1 \neq \tau_2$) one expects to have a function $\phi(\tau_1, \tau_2)$ characterizing a failure or "cavitation curve" such that when $\phi(\tau_1, \tau_2) < 0$ the body deforms homogeneously, when $\phi(\tau_1, \tau_2) = 0$ an infinitesimal (not-necessarily-spherical) void is nucleated, and when $\phi(\tau_1, \tau_2) > 0$ the cavity expands; in order to conform with the symmetric case, one must have $\phi(\tau_{cr}, \tau_{cr}) = 0$. Determining this cavitation curve is the focus of the studies described in this part.

First, we showed that the phenomenon of cavitation is always preceded by a different type of instability if the prescribed loading is dead, and therefore, in all subsequent work we restricted attention on the case of prescribed displacement or prescribed true stress. Next, we determined an exact analytical expression for the cavitation function $\phi$ for a special compressible elastic material. We then developed an analytical procedure which led to an approximate

Reference: R. Abeyaratne and H-s. Hou, On the occurrence of the cavitation instability relative to the asymmetric instability under symmetric dead load conditions, submitted to Quarterly Journal of Mechanics and Applied Mathematics, April 1990.

Cavitation and void collapse under plane strain bi-axial loading in a special compressible elastic material:

Consider a state of plane strain of an infinite medium surrounding a traction-free circular hole, and suppose that the medium is composed of the power-law material considered by Varley and Cumberbatch(1980): in such a material, the nominal stress $\sigma$ and the axial stretch $\lambda$ are related, in plane strain uni-axial tension, by

$$\sigma = (E/k)\ (\lambda^{k-1} - \lambda^{-1}),$$

where E and k are material constants. At infinity, the body is subjected to a state of bi-axial stress

$$\sigma_{11} \to \sigma_1, \quad \sigma_{22} \to \sigma_2, \quad \sigma_{12} \text{ and } \sigma_{21} \to 0.$$

We have analyzed the associated problem and determined the resulting deformation and stress fields. Suppose for example that the hardening exponent has the value k=2. Figure 7 shows a sketch of the $(\sigma_1,\sigma_2)$-plane. The two bold curves in the figure form a critical boundary. If the point $(\sigma_1,\sigma_2)$ corresponding to the remotely applied stress lies in the hatched region, the cavity is open in the deformed configuration and has an elliptical shape. As the point $(\sigma_1,\sigma_2)$ approaches one of the boundaries of the hatched region, the cavity collapses into a crack as shown.

Suppose instead that the hardening exponent in the constitutive law has the value k=1. In this case the cavity is generally ''peanut-shaped''in the deformed configuration as shown in Figure 8. As $(\sigma_1,\sigma_2)$ approaches either of the two bold lines with positive slope, the cavity collapses partially by contacting along part of its boundary as shown in the figure. On the other hand when $(\sigma_1,\sigma_2)$ approaches the bold line with negative slope, the deformed cavity size becomes infinite corresponding to cavitation.

Reference: R. Abeyaratne and H-s. Hou, Void collapse in an elastic solid, accepted for publication in the Journal of Elasticity, July 1989.

Analytical approximation to the cavitation curve:

Consider an infinite incompressible medium, subjected at infinity to an axi-symmetric stress state $(\tau_1, \tau_2, \tau_2)$. We wish to determine the resulting deformation of this body. One deformation that the body can undergo, no matter what the values of $\tau_1$ and $\tau_2$, is a pure homogeneous one. Determining other possible deformations, in exact analytical closed form, is a difficult task; consequently, in this part of the present study, we attempted to find an approximate analytical deformation field.

Consider the sub-class, say 'A', of all kinematically admissible deforma-

tions which are of the form

$$\left.\begin{aligned} y_1 &= f(r)x_1, \\ y_2 &= g(r)x_2, \\ y_3 &= g(r)x_3; \end{aligned}\right\} \tag{1}$$

here $(x_1, x_2, x_3)$ are the coordinates of a particle before deformation and $(y_1, y_2, y_3)$ are its coordinates in the deformed configuration, $r^2 = x_1^2+x_2^2+x_3^2$, and f and g are positive-valued functions. Note that deformations in this class possess axi-symmetry; moreover, the special case f=g corresponds to radially symmetric deformations, while the special case f=constant, g=constant describes a homogeneous axi-symmetric deformation. In view of the incompressibility of the material one can show that deformations in 'A' must necessarily have the form

$$\left.\begin{aligned} y_1 &= \lambda^{-2}\,(1 + \beta^3/R^3)^{1/3}\, x_1, \\ y_2 &= \lambda\,(1 + \beta^3/R^3)^{1/3}\, x_2, \\ y_3 &= \lambda\,(1 + \beta^3/R^3)^{1/3}\, x_3, \end{aligned}\right\} \tag{2}$$

where $\lambda > 0$ and $\beta \geq 0$ are kinematically undetermined constants. When $\beta=0$ this describes a pure homogeneous deformation, while when $\lambda=1$ it corresponds to a radial deformation involving a cavity of current radius $\beta$. In general, such a deformation carries the origin in the undeformed configuration into an ellipsoidal cavity; $\beta$ is a measure of the size of the cavity while $\lambda$ describes its departure from spherical symmetry.

Thus, the collection of deformations 'A' corresponds to a two-parameter family of deformations and the "best" values for the parameters $\lambda$, $\beta$ may be found by minimizing the appropriate total energy of the body. Such a calculation leads to the following two equations relating the stresses $\tau_1$ and $\tau_2$ to the geometric parameters $\lambda$ and $\beta$:

$$\left.\begin{aligned} \tau_1 + 2\tau_2 &= 3/(4\pi\beta^2)\ \partial E/\partial\beta, \\ \tau_2 - \tau_1 &= 3\lambda/(8\pi(1+\beta^3))\ \partial E/\partial\lambda; \end{aligned}\right\} \tag{3}$$

here $E(\beta,\lambda)$ is the total strain energy stored in the body corresponding to the deformation (2). At the instant of cavitation one has $\beta=0$; setting $\beta=0$ in (3) and then eliminating (in principle) the parameter $\lambda$ between those two equations, leads to a relationship between $\tau_1$ and $\tau_2$ which is the approximate equation of the cavitation curve $\phi(\tau_1,\tau_2)=0$. The curve in Figure 9 shows the result for the neo-Hookean material. This same procedure can be carried out for elastic-plastic materials as well, provided the principle of virtual work is used in place of extremizing the total energy. The curves in Figure 10 show the result for a piecewise power-law material for different values of the hardening exponent.

Reference: H-s. Hou and R. Abeyaratne, Axi-symmetric cavitation in solids, manuscript in preparation.

### Numerical verification of accuracy of approximate cavitation curves:

In order to investigate the accuracy of the preceding analytical approximation, we undertook a numerical study. By using the large strain analysis capabilities of the finite element program ABAQUS, we have carried out numerical simulations for both the neo-Hookean material and piecewise power-law elastic-plastic materials. For computational purposes, we must, of course, deal with finite sized regions and finite sized cavities. Thus, we have taken a "large" body containing a "small" cavity and incremented the stress in steps (along radial lines in stress space) and solved for the various fields including the cavity dimensions. In a graph of cavity "size" (= one half the sum of major and minor axes) versus stress, the cavity size is found to growth very slowly first and then to suddenly grow rapidly. We determined the limit load, i.e. the maximum value of applied stress, and took it to be an approximation for the stress level at cavitation.

The circles in Figure 9 show the cavitation curve for the neo-Hookean material as determined by the finite element solution. We see that the analytical solution is a good approximation and it certainly shows the correct trends. Figure 11 shows the analytical and numerical cavitation curves (corresponding to the curve and the circles respectively) for an elastic-perfectly plastic material; the circles in that figure were taken from the numerical solution of Hutchinson et al.(1989). Similar comparisons for materials with strain hardening were also carried out but are not shown here. Finally Figure 12 shows a comparison between the actual deformation fields; the two curves shown are the deformed cavity shape according to (2) at two different values of load, while the circles correspond to the finite element solution; the figure is drawn for the neo-Hookean material.

Reference: H-s. Hou and R. Abeyaratne, Axi-symmetric cavitation in solids, manuscript in preparation.

### Application of cavitation curves: comparison of theory with experiment.

We have used the failure curves $\phi$ generated by the aforementioned studies, and applied it to two experimental situations: one, the experiments of Gent and Lindley(1958) on thin rubber disks, where depending on the thickness of the disk, they either observed or did not observe cavitation, and two, the experiments of Ashby et al.(1989) on highly constrained lead wires. The predictions of the model have been consistent with the observations. In Gent and Lindley's experiments (see description previously) they studied cavitation in rubber disks of various thicknesses. A finite element stress analysis of different specimens, as the load is increased, yields trajectories in stress space corresponding to the evolution of the stress at the center of the specimen, as shown in Figure 13. Cavitation occurs when the stress trajectory intersects the cavitation curve. The figure shows that the two thickest specimens do not intersect this curve, and therefore do not cavitate at the center of the specimen, while the others do. This observation, as well as the values of load at which cavitation does occur in the thinner specimens, agrees with Gent and Lindley's observations.

Reference: H-s. Hou and R. Abeyaratne, Axi-symmetric cavitation in solids, manuscript in preparation.

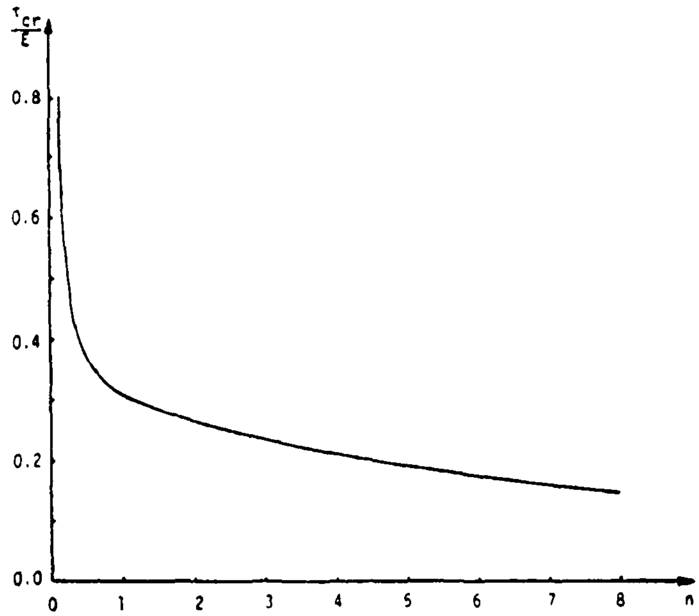


Figure 1. Variation of critical stress $\tau_{cr}$ with hardening exponent n at fixed Poisson's ratio $\nu$=0.3.

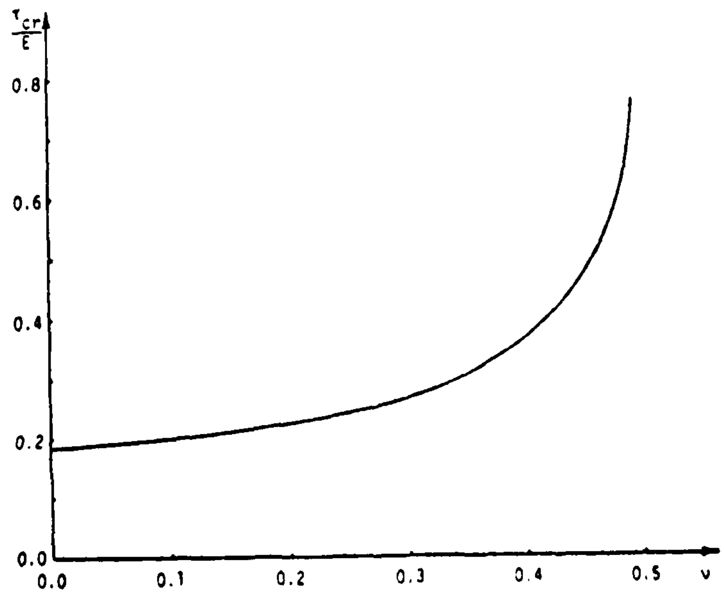


Figure 2. Variation of critical stress $\tau_{cr}$ with Poisson's ratio $\nu$ at fixed hardening exponent n=2.

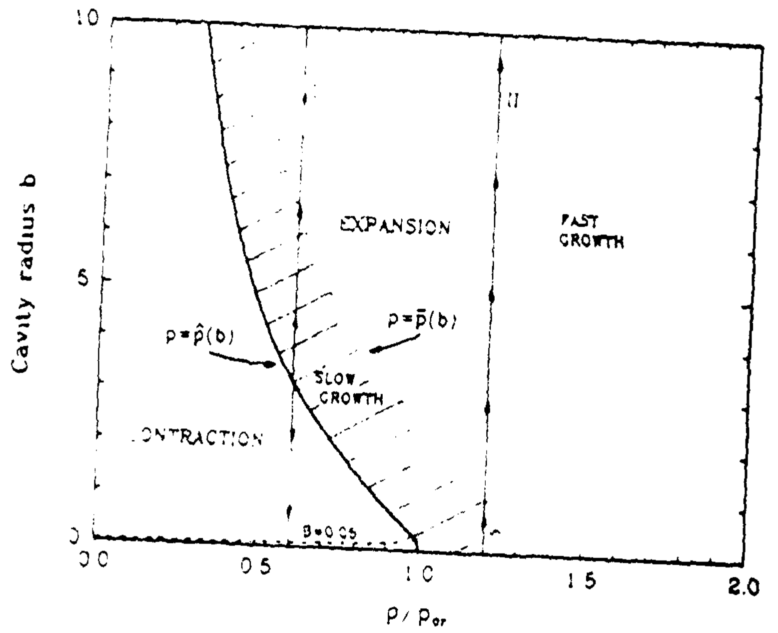


Figure 3. Void expansion and contraction domains for rate-dependent material.

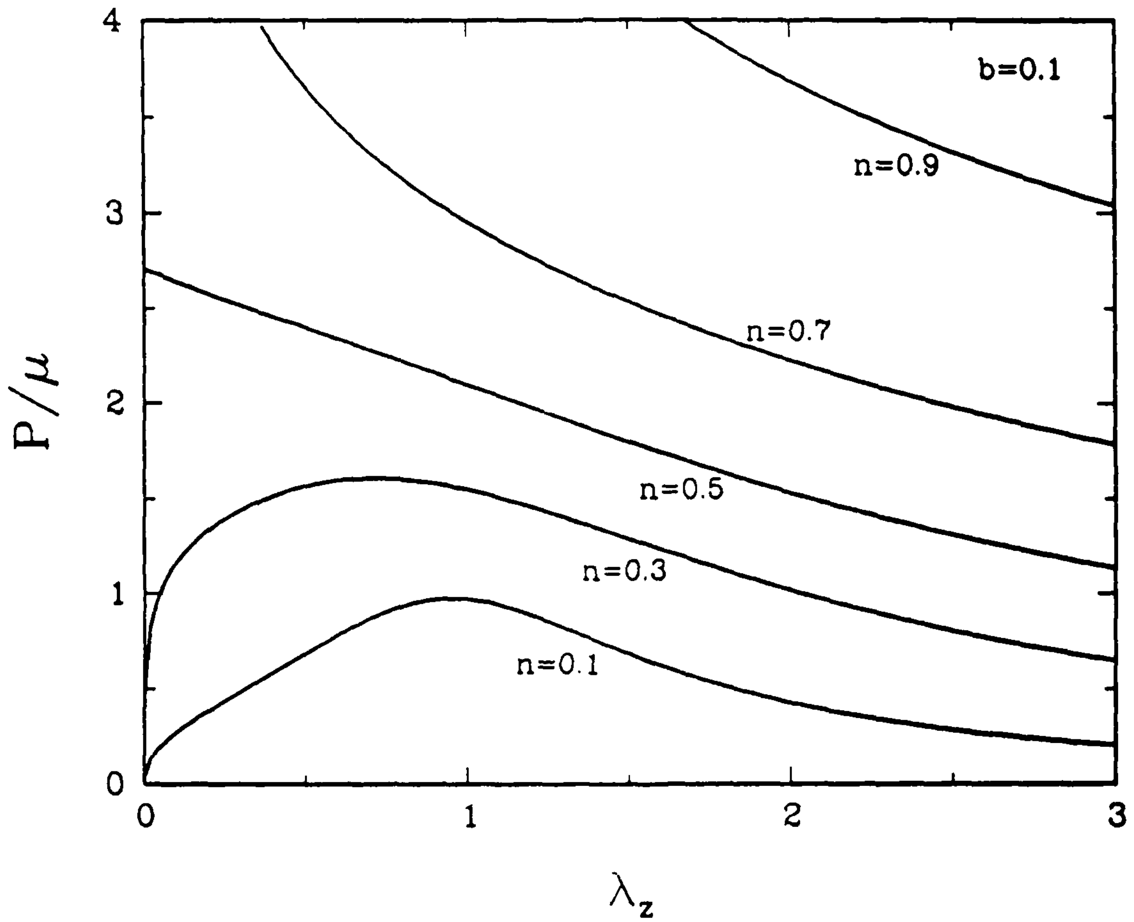


Figure 4. Cavitation curves $f(p,\lambda_z)=0$ for power-law material for different hardening exponents.

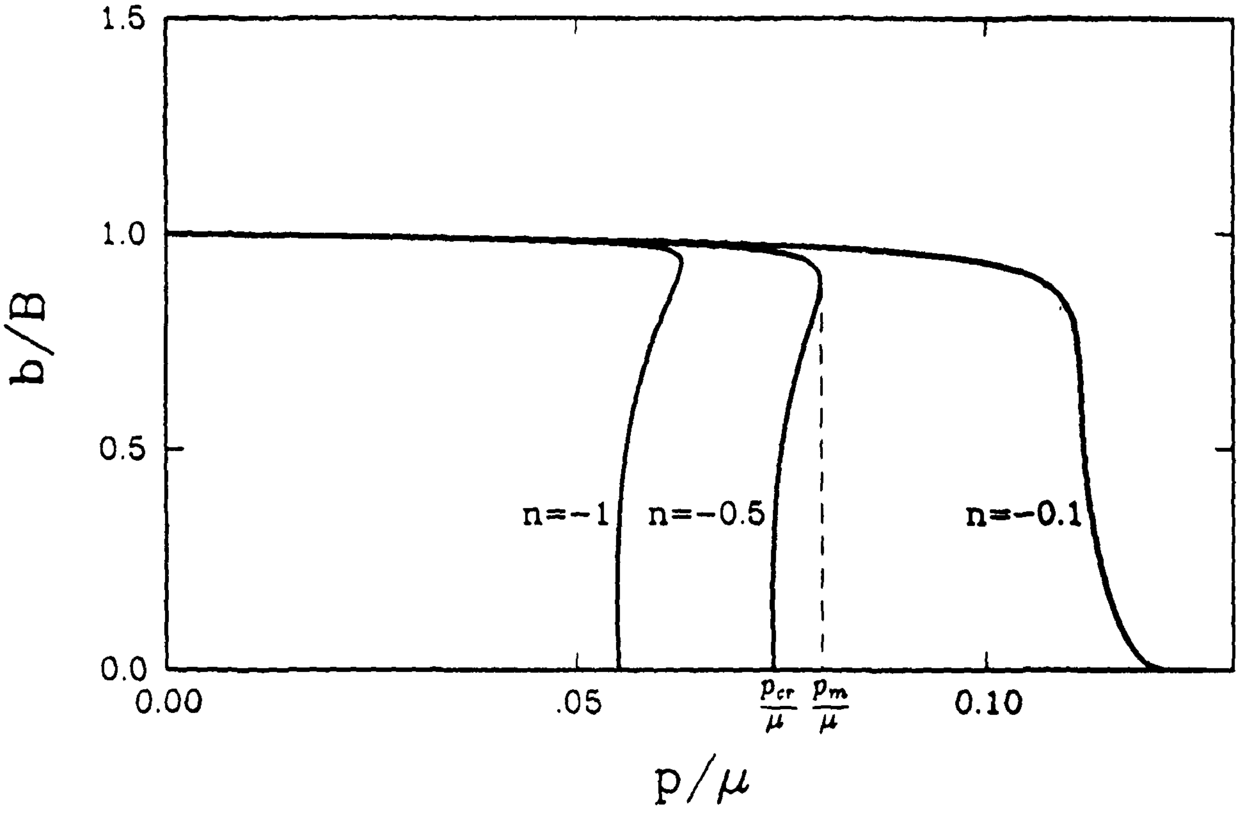


Figure 5. Variation of the void radius b with the applied pressure p for a spherical shell for different hardening exponents n.

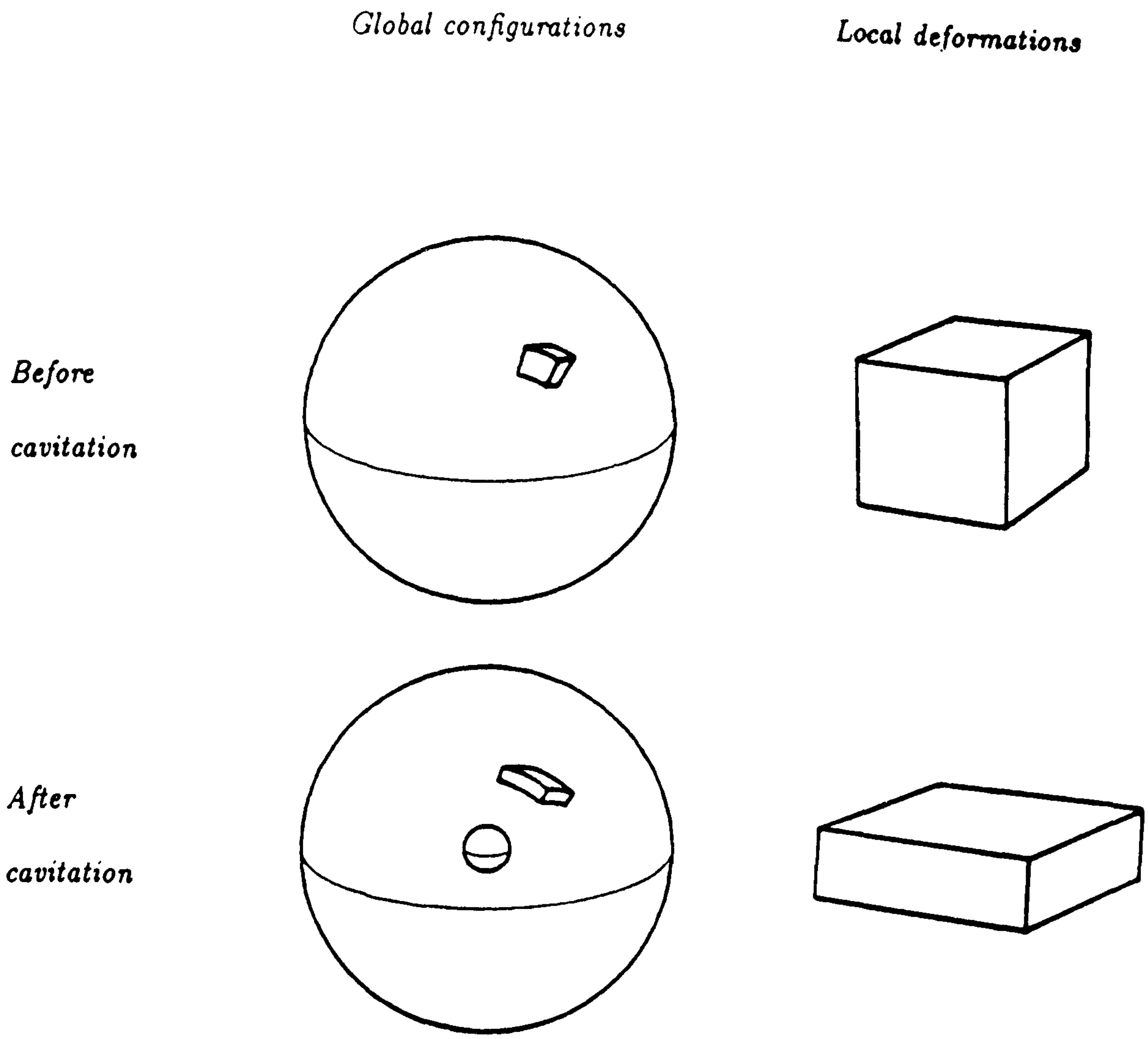


Figure 6. Pre- and post-bifurcation sketches of a material element in a cavitating sphere.

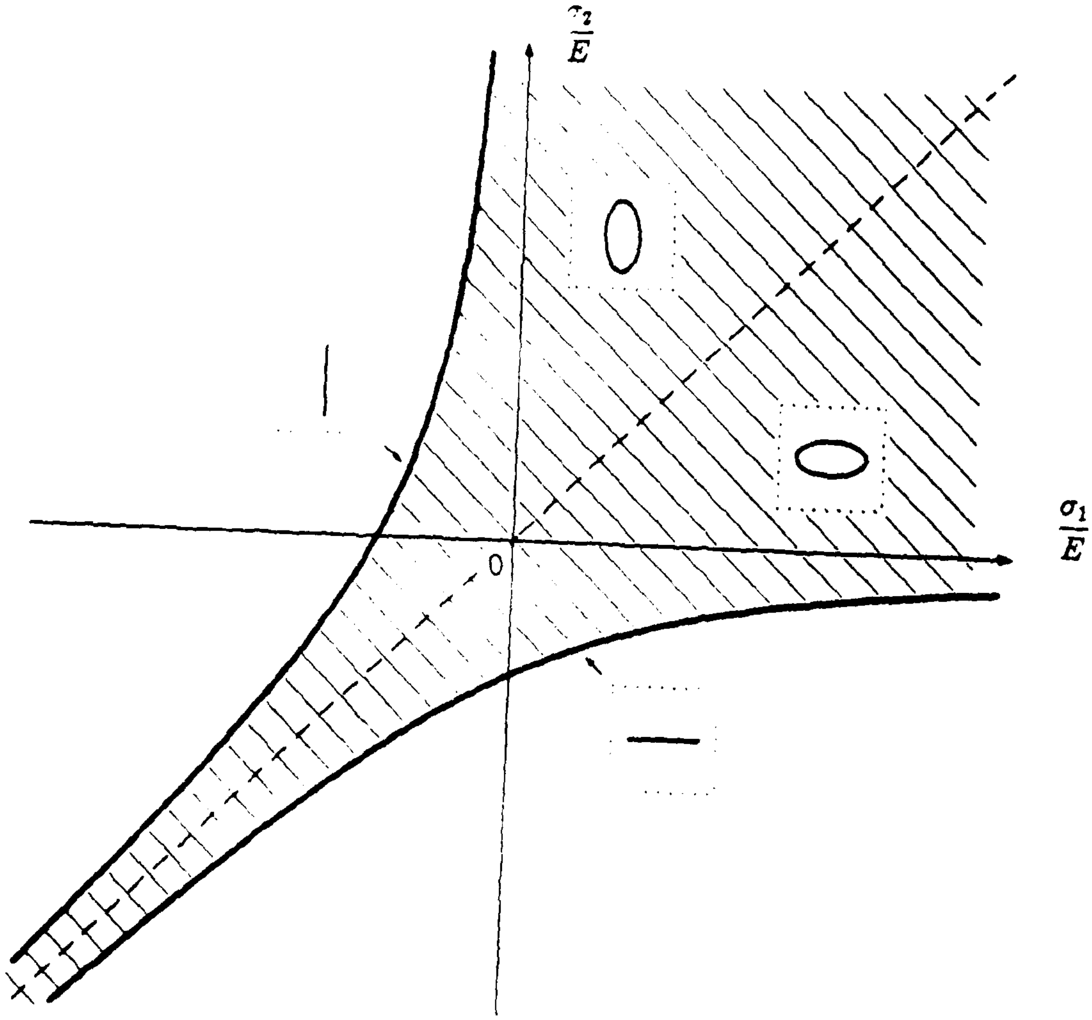


Figure 7. Domain of $(\sigma_1, \sigma_2)$–plane where cavity is open.
Varley and Cumberbatch material with k=2.

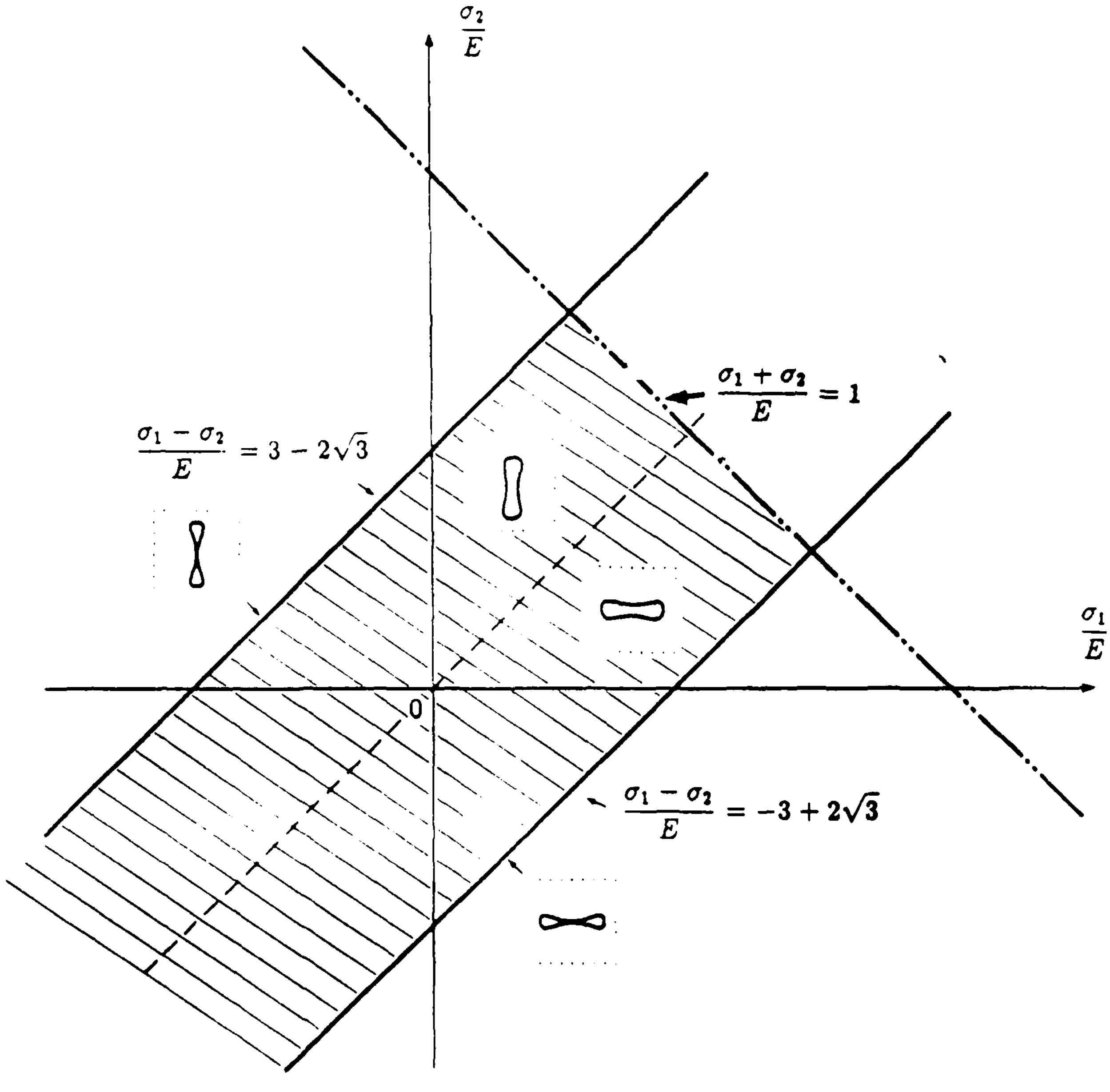


Figure 8. Domain of $(\sigma_1, \sigma_2)$–plane where cavity is open.
Valey and Cumberbatch material with k=1.

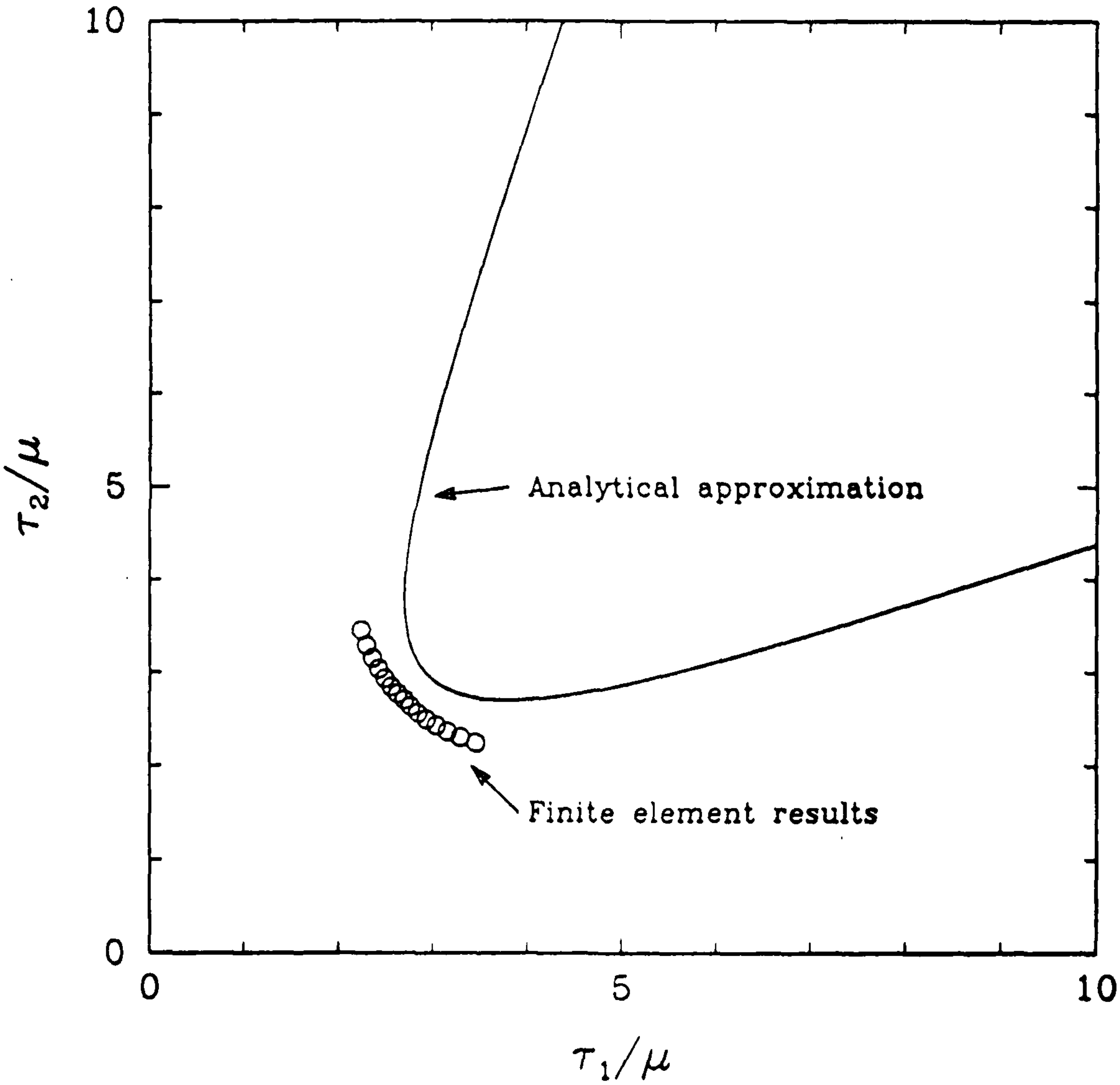


Figure 9. Cavitation curve $\phi(\tau_1,\tau_2)=0$ for neo-Hookean material.

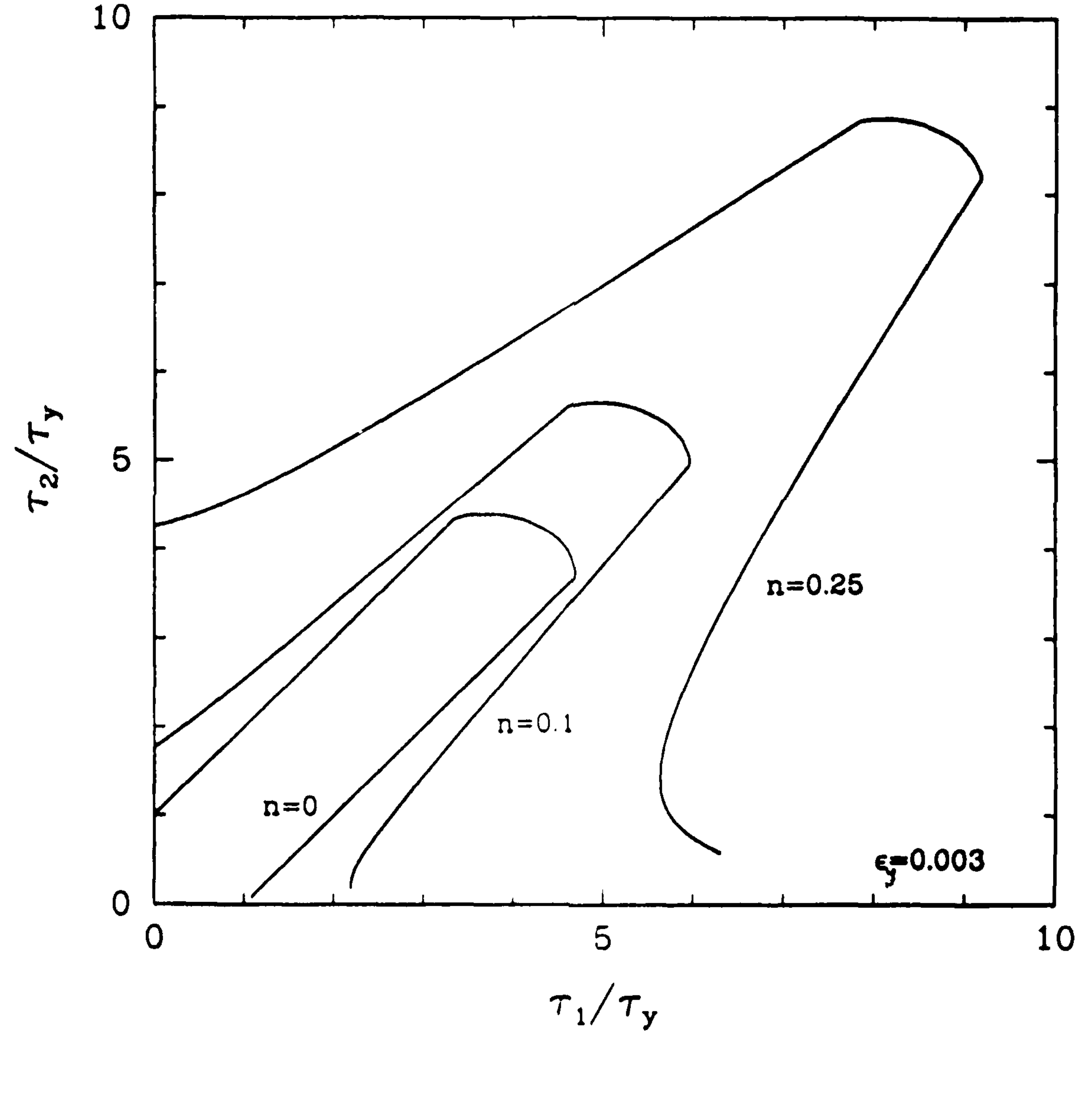


Figure 10. Cavitation curve $\phi(\tau_1, \tau_2)=0$ for elastic-plastic materials with different hardening.

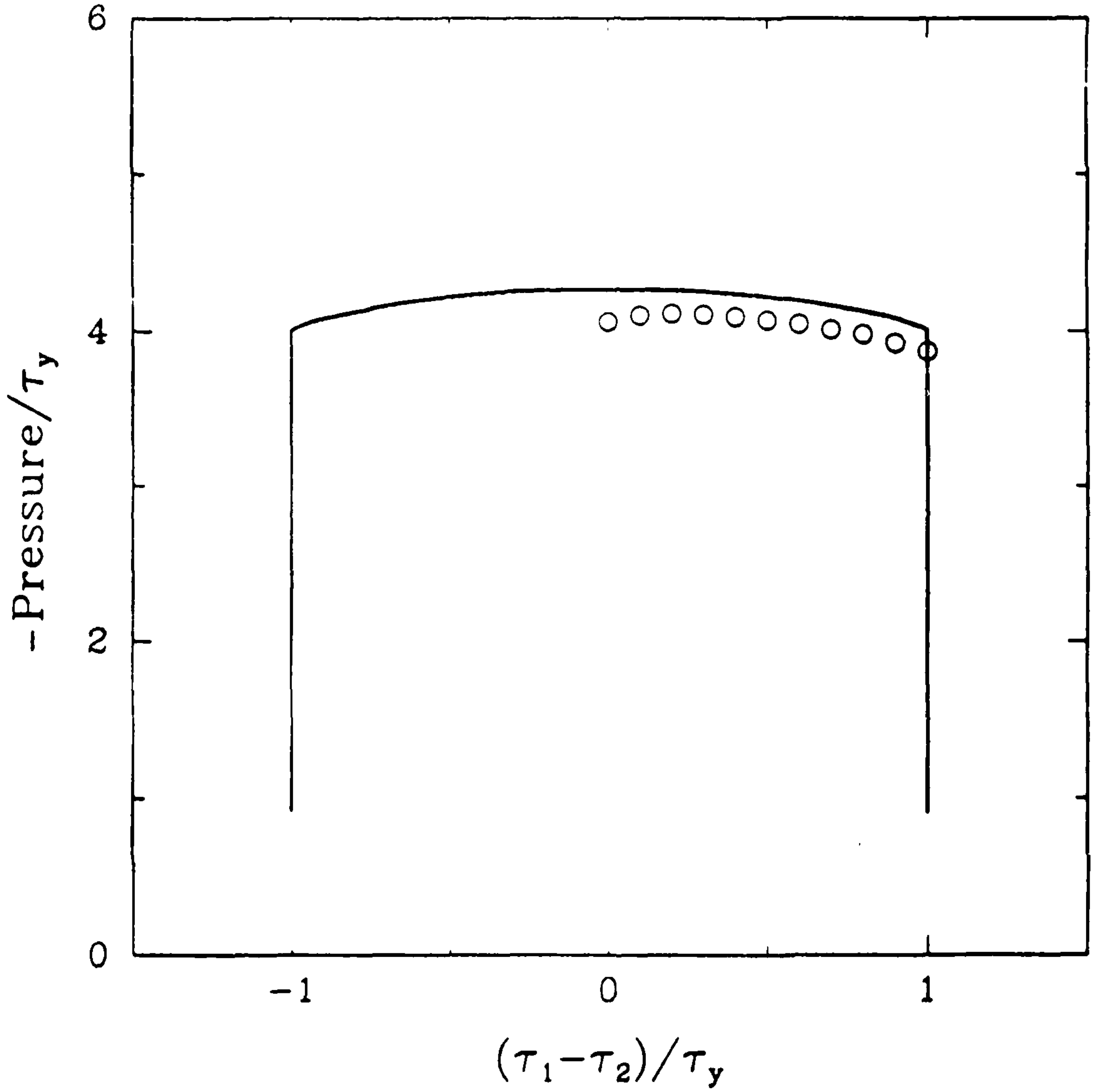


Figure 11. Cavitation curve $\phi(\tau_1,\tau_2)=0$ for elastic-perfectly plastic material.

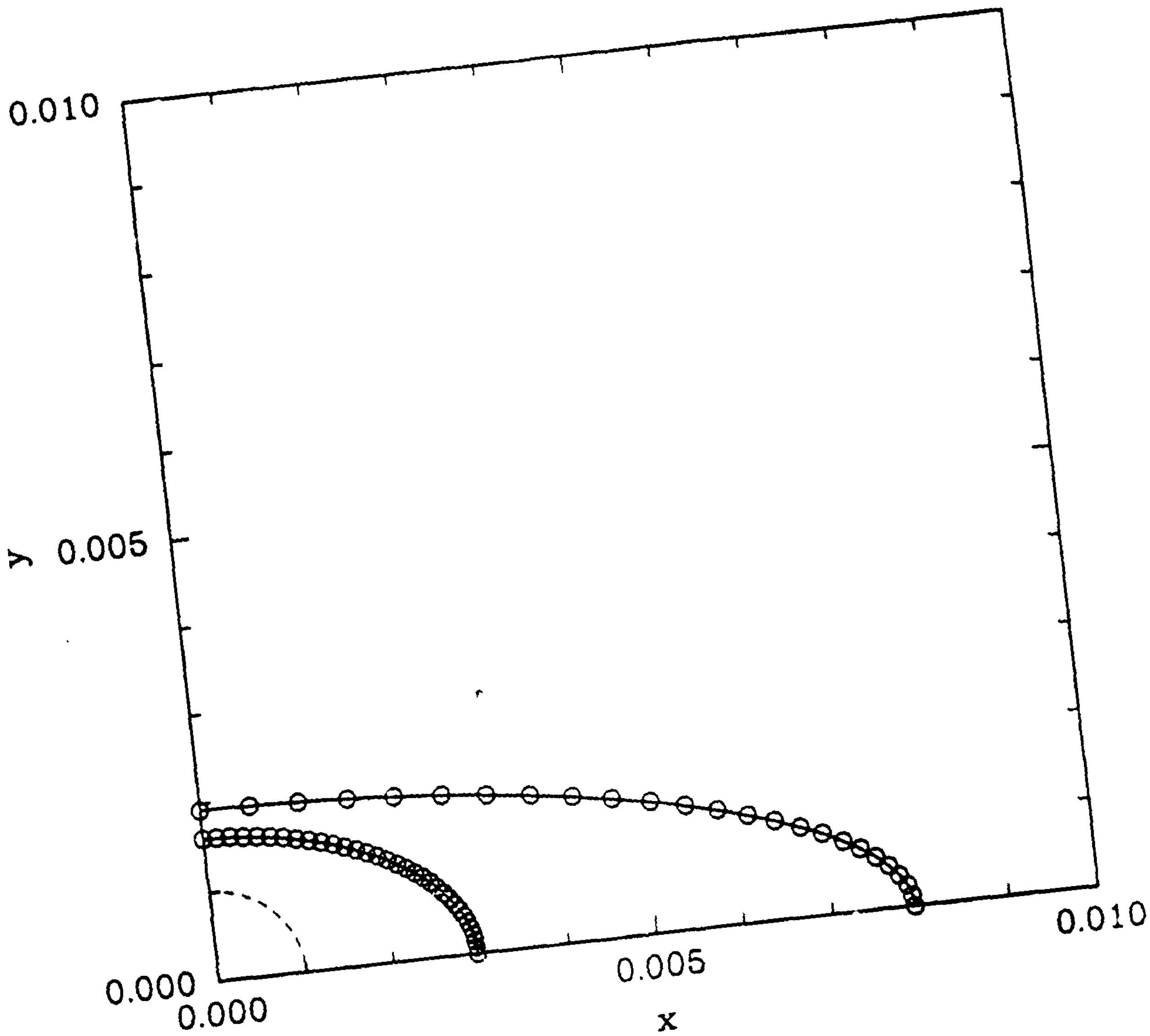


Figure 12. Deformed shape of the cavity according to approximate and numerical solutions.

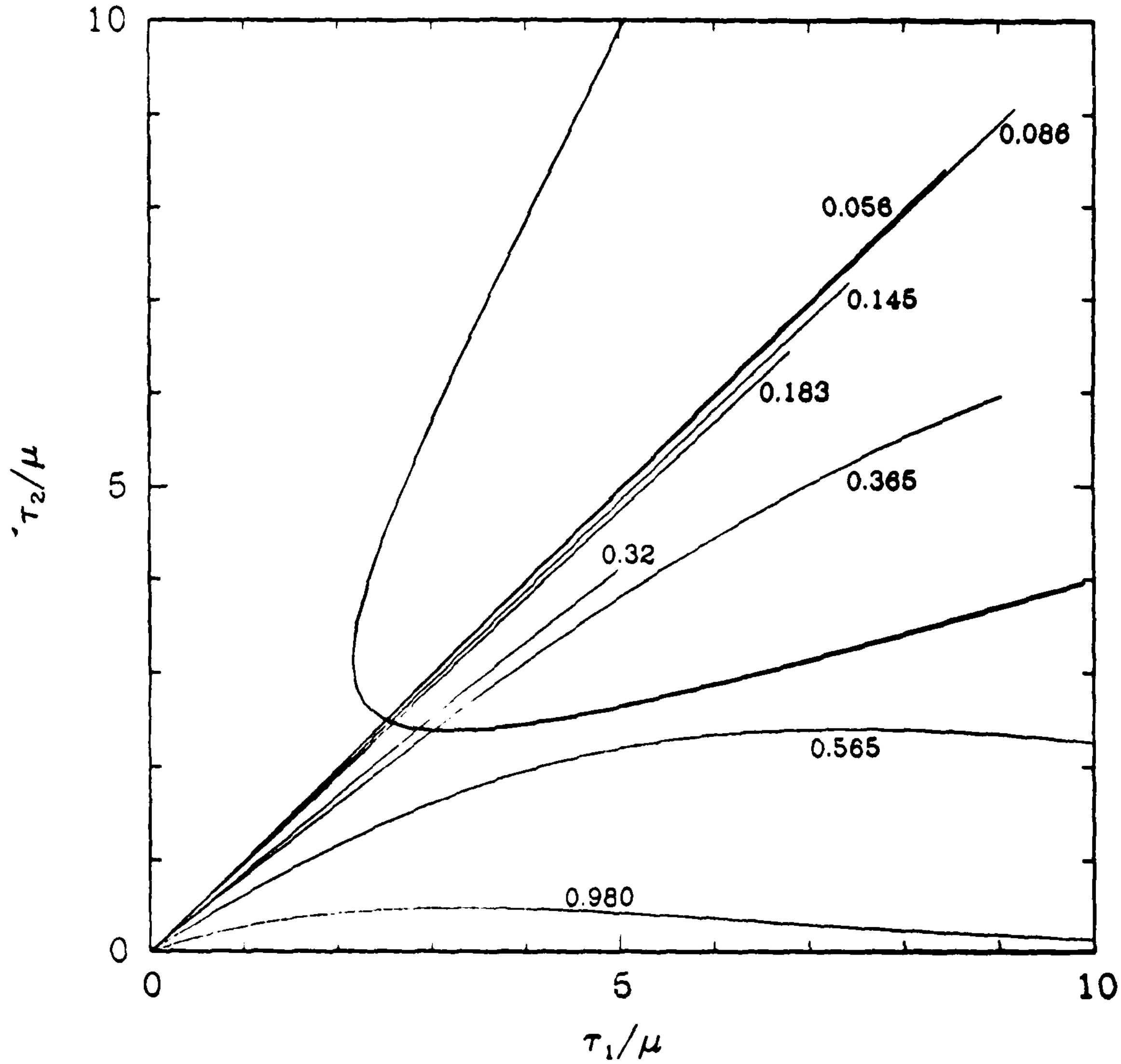


Figure 13. The intersection of stress trajectories and the cavitation curve for rubber disks of various thicknesses.